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Derivative corrections from noncommutativity

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Abstract: We show that an infinite subset of the higher-derivative α' corrections to the DBI and Chern-Simons actions of ordinary commutative open-string theory can be determined using noncommutativity. Our predictions are compared to some lowest order α' corrections that have been computed explicitly by Wyllard (hep-th/0008125), and shown to agree.

Keywords: Superstrings and Heterotic Strings, D-branes, Non-Commutative Geometry.

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1. Introduction

Noncommutativity has provided important new insights into the nature of string theory [1, 2]. In the presence of a 2-form B field, one has the option to describe effective actions for open strings in either commutative or noncommutative descriptions. Using the continuous "description parameter" Φ introduced in [2], one can actually interpolate between the two types of descriptions, with $\Phi = B$ representing the commutative theory. The other useful choices are $\Phi = 0$ (which arises naturally in the point-splitting regularization) and $\Phi = -B$ (which has been called the "background-independent" description [2, 3], closely related to matrix theory). Depending on the question that one wants to address, one can choose any of these descriptions for convenience.

In different descriptions, the natural low-energy limits are also different. The parameter governing higher-derivative corrections in string theory is α', and one can take this to zero keeping fixed various different quantities. An important limit in the noncommutative description is the Seiberg-Witten limit, $\alpha' \to 0$ keeping fixed the open-string metric G along with the 2-form field B and the open-string coupling G_s. In this limit, derivative corrections to the noncommutative actions (DBI and Chern-Simons) vanish.

It was shown in ref. [2] that the noncommutative DBI action is equivalent to the commutative one upto total derivative terms. In recent times, it has been understood that exact equivalence (not just upto total derivatives) of commutative and noncommutative actions can be obtained if one additionally inserts an open Wilson line [4]–[7] on the noncommutative side. This has been useful in writing down gauge-invariant couplings of open string modes to closed-string NSNS fields [8, 9] and RR fields [10]–[13]. This has provided a powerful tool to extract new information. For example it allows one to obtain an exact expression for the Seiberg-Witten map between commutative and noncommutative gauge couplings in the abelian case.

In this note we exhibit a new application of the noncommutative description of string theory. We start by assuming exact equivalence of the commutative and noncommutative actions, including all derivative corrections on both sides. It is important to note that terms which, for constant backgrounds, would have been total derivatives, are also retained. Next we take the Seiberg-Witten limit, which sets to zero the α' corrections on the noncommmutative side, and reduces it to a sum of Yang-Mills and Chern-Simons actions. Comparison of the two sides now yields definite predictions for the derivative corrections on the commutative side, or at least those corrections (there are infinitely many) that survive the Seiberg-Witten limit.

Earlier attempts to study derivative corrections to the DBI action using noncommutativity can be found in ref. [14]. The principal new ingredient in our work is the fact that with open Wilson lines, one has exact agreement between commutative and noncommutative actions, including couplings to closed-string backgrounds at nonzero momentum. This allows us to make very explicit predictions and compare them with perturbative string amplitudes.

Some of the derivative corrections in open-string theory were computed explicitly in recent times [15] in the boundary-state formalism.[1] We will show in a number of cases that the numerical coefficients and index structures given by these computations can be reproduced using our arguments, by expanding the n-ary product $*_n$ that has recently played an important role in the noncommutative description of string effective actions.

The agreement between the predictions of noncommutativity and the computations of ref. [15] might seem somewhat fortuitous, given that there is always a freedom of field redefinitions. We will comment on this point in some detail in the conclusions.

2. Background and proposal

In what follows, we will always work with the BPS D9-branes of type IIB string theory, though the discussion can in principle be extended to lower D-branes. The

[1] Earlier work on the computation of such corrections can be found in ref. [16]. We use the results of ref. [15] as they are the most complete to date, and also because the choice of field variables turns out to have a special significance, as we will see.

DBI and Chern-Simons actions on the brane in the commutative description are:

$$\begin{aligned} S_{DBI} &= \frac{1}{g_s} \int \sqrt{g + 2\pi\alpha'(B+F)} \\ S_{CS} &= \frac{1}{g_s} \int \sum_n C^{(n)} \, e^{2\pi\alpha'(B+F)} \, . \end{aligned} \tag{2.1}$$

In the latter expression, the exponential is to be expanded to keep the 10-form part.

The noncommutative description[2] is parametrized by the noncommutativity parameter θ, the open-string metric G_{ij}, the open-string coupling G_s, and a "description parameter" Φ, in terms of which the relationship between closed-string and open-string parameters is given by:

$$\begin{aligned} N^{ij} \equiv \left(\frac{1}{g + 2\pi\alpha' B} \right)^{ij} &= \frac{\theta}{2\pi\alpha'} + \frac{1}{G + 2\pi\alpha'\Phi} \\ \frac{\sqrt{\det(g + 2\pi\alpha' B)}}{g_s} &= \frac{\sqrt{\det(G + 2\pi\alpha'\Phi)}}{G_s} \, . \end{aligned} \tag{2.2}$$

In what follows, it is most convenient to work in the $\Phi = -B$ description, where the contact with matrix theory is explicit. In this description, the DBI action can be written equivalently in two convenient forms:

$$\begin{aligned} \hat{S}_{DBI} &= \frac{1}{G_s} \int \sqrt{G + 2\pi\alpha'(\hat{F} - B)} \\ &= \frac{1}{g_s} \int \frac{\mathrm{Pf}\, Q}{\mathrm{Pf}\, \theta} \sqrt{g + 2\pi\alpha'\, Q^{-1}} \, . \end{aligned} \tag{2.3}$$

Here, G_s, G and θ are the open-string coupling, metric and noncommutativity parameter respectively, defined by

$$\theta^{ij} = (B^{-1})^{ij} \, , \qquad G_{ij} = -(2\pi\alpha')^2 B_{ik}\, g^{kl}\, B_{lj} \, , \qquad G_s = g_s \sqrt{\frac{\det 2\pi\alpha' B}{\det g}} \, , \tag{2.4}$$

while Q^{ij} is given by

$$Q^{ij} = \theta^{ij} - \theta^{ik} \hat{F}_{kl}\, \theta^{lj} \tag{2.5}$$

and Pf denotes the pfaffian or square root of the determinant. We also note the following explicit expression for Q^{-1}, which will be useful later on:

$$Q^{-1} = \theta^{-1} + \hat{F} \frac{1}{1 - \theta\hat{F}} \, . \tag{2.6}$$

As is well-known, the above relations also hold in the Seiberg-Witten limit [2]:

$$\alpha' \to 0, \qquad G, B, G_s \;\; \text{fixed} \tag{2.7}$$

regardless of the description parameter Φ. Since we will be mainly working in this limit in what follows, our results can also be interpreted as being valid in any description. In particular, this observation explains the agreement of the $\Phi = -B$ results of refs. [9] with the explicit string amplitude calculations of refs. [17, 18]. The latter of course involve the point-splitting regularization and therefore they correspond to the $\Phi = 0$ description.

The noncommutative Chern-Simons action for constant fields can be written as follows [10]

$$\hat{S}_{CS} = \frac{1}{g_s} \int \frac{\operatorname{Pf} Q}{\operatorname{Pf} \theta} \sum_n C^{(n)} e^{2\pi\alpha' Q^{-1}} \,, \tag{2.8}$$

where the exponential is to be expanded so that the total form has the rank of the brane worldvolume, namely 10 in our case.

For nonconstant fields, the above actions are not gauge-invariant and one needs to introduce an open Wilson line. This is defined as

$$W(x, C) \equiv \exp\left(-i \int_0^1 d\tau \frac{\partial \xi^i(\tau)}{\partial \tau} \hat{A}_i(x + \xi(\tau))\right), \tag{2.9}$$

where the contour of the Wilson line is defined in terms of a fixed momentum k by $\xi^i(\tau) = \theta^{ij} k_j \, \tau$ with $0 \leq \tau \leq 1$. This operator must be inserted to make the coupling to a closed-string mode of momentum k gauge-invariant. For example, if we consider the linearized coupling to a dilaton $\widetilde{D}(k)$, the DBI action must be replaced by[8, 9]

$$\hat{S}_{DBI}(k) = \frac{\widetilde{D}(-k)}{g_s} \int L_* \left\{ \frac{\operatorname{Pf} Q}{\operatorname{Pf} \theta} \sqrt{g + 2\pi\alpha' \, Q^{-1}} \, W(k, C) \right\} * e^{ik.x} \,. \tag{2.10}$$

The operation L_* consists of smearing all operators along the contour of the Wilson line and path-ordering the resulting expression with respect to the noncommutative $*$ product.

In the same way, the coupling of open-string modes to a Ramond-Ramond form of nonzero momentum is given by [11, 12, 13]

$$\sum_n \widetilde{C}^{(n)}(-k) \int L_* \left\{ \frac{\operatorname{Pf} Q}{\operatorname{Pf} \theta} \, e^{2\pi\alpha' Q^{-1}} \, W(x, C) \right\} * e^{ik.x} \,, \tag{2.11}$$

where again we need to pick out the 10-form contributions in the above expression.

As explained in refs. [19, 8], the expansion of the above expressions can be written in terms of an n-ary product called $*_n$, which maps a collection of n functions $f_1, f_2, \ldots, f_n$ to a single function that we denote $\langle f_1, f_2, \ldots, f_n \rangle_{*_n}$. The definition of $*_2$ is relatively simple:

$$\langle f(x), g(x) \rangle_{*_2} \equiv f(x) \frac{\sin(\frac{1}{2} \overleftarrow{\partial_p} \, \theta^{pq} \, \overrightarrow{\partial_q})}{\frac{1}{2} \overleftarrow{\partial_p} \, \theta^{pq} \, \overrightarrow{\partial_q}} g(x) \,. \tag{2.12}$$

More information about the role of $*_n$ products, and general formulae, can be found in refs. [20, 19, 8, 9, 21].

More specifically, expanding out an L_* product leads to the Fourier transform of the $*_n$ products. At this point it is often more convenient to go back to position space. Hence in what follows, we will usually work in position space, but will be forced to use momentum space whenever the L_* product is yet to be expanded out. We hope this will be clear from the context.

Now let us summarize the basic approach of this paper. The DBI and CS actions written here will in general have corrections that involve higher powers of α'. Let us denote these corrections by $\Delta\hat{S}_{DBI}$ and $\Delta\hat{S}_{CS}$ respectively. The requirement that noncommutative and commutative actions are really the same means that

$$\begin{aligned} S_{DBI} + \Delta S_{DBI} &= \hat{S}_{DBI} + \Delta\hat{S}_{DBI} \\ S_{CS} + \Delta S_{CS} &= \hat{S}_{CS} + \Delta\hat{S}_{CS} \, . \end{aligned} \tag{2.13}$$

Here the terms on the left hand side are the open-string effective actions plus their derivative corrections in the usual commutative description.

Note that in $\Delta\hat{S}_{DBI}$ and $\Delta\hat{S}_{CS}$, indices are always contracted with the open string metric G_{ij}. Therefore in the Seiberg-Witten limit all these noncommutative corrections vanish, and the identities in eq. (2.13) reduce to

$$\begin{aligned} S_{DBI}\Big|_{SW} + \Delta S_{DBI}\Big|_{SW} &= \hat{S}_{DBI}\Big|_{SW} \\ S_{CS}\Big|_{SW} + \Delta S_{CS}\Big|_{SW} &= \hat{S}_{CS}\Big|_{SW} \, , \end{aligned} \tag{2.14}$$

where the subscript SW indicates that the Seiberg-Witten limit has been taken.

In what follows, our strategy will be to derive information about $\Delta S_{DBI}\big|_{SW}$ and $\Delta S_{CS}\big|_{SW}$ using the exact knowledge of the commutative and noncommutative DBI and Chern-Simons actions. Some terms in ΔS_{DBI} and ΔS_{CS} have been computed in ref. [15] and we will compare them in the SW limit with the prediction from the r.h.s., finding complete agreement. We will discuss to what extent this allows us to recover full information about these terms away from the SW limit.

3. The Dirac-Born-Infeld action

In this section we wish to compare the sum of the commutative DBI action S_{DBI} plus the derivative corrections to it ΔS_{DBI} (some of which are computed in ref. [15]) with the noncommutative DBI action $\hat{S}_{DBI}$, after taking the Seiberg-Witten limit on both sides.

3.1 Dilaton coupling, order F^2

The dilaton couples to the entire lagrangian density, so we need to consider the full DBI action. We will start by restricting to terms quadratic in F. To this order, we have:

$$S_{DBI} = \int \frac{\sqrt{\det(g+2\pi\alpha' B)}}{g_s}\Bigg[1 + \frac{2\pi\alpha'}{2}\,\mathrm{tr}(NF) - \frac{(2\pi\alpha')^2}{4}\,\mathrm{tr}(NFNF) + \\ + \frac{(2\pi\alpha')^2}{8}\big(\,\mathrm{tr}\,NF\big)^2 + \cdots\Bigg]. \tag{3.1}$$

In the Seiberg-Witten limit we have $N^{ij} \to \theta^{ij}/2\pi\alpha'$ and therefore:

$$S_{DBI}\Big|_{SW} = \int \frac{\sqrt{\det(g+2\pi\alpha' B)}}{g_s}\left[1+\frac{1}{2}\,\mathrm{tr}(\theta F)-\frac{1}{4}\,\mathrm{tr}(\theta F\theta F)+\frac{1}{8}(\,\mathrm{tr}\,\theta F)^2+\cdots\right]. \tag{3.2}$$

Note that, here and later in the paper, we insert this limit only in the bracketed series expansion, leaving the prefactor untouched. This is because the prefactor will eventually cancel with the corresponding prefactor on the noncommutative side when we compare the two.

Let us now convert the commutative field strengths F appearing in this expression into noncommutative field strengths $\hat{F}$, using the Seiberg-Witten map. To the order that we need it, this map is:

$$F_{ab} = \hat{F}_{ab} + \theta^{kl}\left(\langle \hat{A}_k, \partial_l \hat{F}_{ab}\rangle_{*_2} - \langle \hat{F}_{ak}, \hat{F}_{bl}\rangle_{*_2}\right), \tag{3.3}$$

where

$$\hat{F}_{ab} = \partial_a \hat{A}_b - \partial_b \hat{A}_a + \theta^{kl}\langle \partial_k \hat{A}_a, \partial_l \hat{A}_b\rangle_{*_2}\,. \tag{3.4}$$

Here we have used an identity relating the Moyal $*$ commutator and the $*_2$ product:

$$-i[f,g]_* = \theta^{ij}\langle \partial_i f, \partial_j g\rangle_{*_2}\,. \tag{3.5}$$

Inserting the Seiberg-Witten map into eq. (3.2), we find

$$S_{DBI}\Big|_{SW} = \int \frac{\sqrt{\det(g+2\pi\alpha' B)}}{g_s}\Bigg[1 + \theta^{ij}\partial_j \hat{A}_i + \frac{1}{2}\theta^{ba}\theta^{kl}\langle \partial_k \hat{A}_a, \partial_l \hat{A}_b\rangle_{*_2} + \\ + \frac{1}{2}\theta^{ab}\theta^{kl}\left(\langle \hat{A}_k, \partial_l \hat{F}_{ab}\rangle_{*_2} - \langle \hat{F}_{ak}, \hat{F}_{bl}\rangle_{*_2}\right) - \\ - \frac{1}{4}\theta^{ij}\theta^{kl}\hat{F}_{jk}\hat{F}_{li} + \frac{1}{8}\left(\theta^{ij}\hat{F}_{ij}\right)^2\Bigg]. \tag{3.6}$$

Some manipulation of the last few terms permits us to rewrite this as:

$$S_{DBI}\Big|_{SW} = \int \frac{\sqrt{\det(g+2\pi\alpha' B)}}{g_s}\Bigg[1 + \theta^{ij}\partial_j \hat{A}_i + \frac{1}{2}\theta^{ij}\theta^{kl}\langle \partial_j \hat{A}_k, \partial_l \hat{A}_i\rangle_{*_2} + \\ + \frac{1}{2}\theta^{ba}\theta^{kl}\langle \hat{A}_k, \partial_l \hat{F}_{ab}\rangle_{*_2} + \frac{1}{4}\theta^{ij}\theta^{kl}\left(\langle \hat{F}_{jk}, \hat{F}_{li}\rangle_{*_2} - \hat{F}_{jk}\hat{F}_{li}\right) + \\ + \frac{1}{8}\theta^{ij}\theta^{kl}\langle \hat{F}_{ji}, \hat{F}_{lk}\rangle_{*_2} - \frac{1}{8}\theta^{ij}\theta^{kl}\left(\langle \hat{F}_{ji}, \hat{F}_{lk}\rangle_{*_2} - \hat{F}_{ji}\hat{F}_{lk}\right)\Bigg] \tag{3.7}$$

which is the form in which it will be useful.

Let us now turn to the noncommutative side. Here, we only need to keep the terms arising from expansion of the Wilson line, since all other terms are suppressed by powers of α' in the Seiberg-Witten limit. The Wilson line gives us:

$$\hat{S}_{DBI}\Big|_{SW} = \int \frac{\sqrt{\det(G+2\pi\alpha'\Phi)}}{G_s}\left[1+\theta^{ij}\partial_j\hat{A}_i+\frac{1}{2}\theta^{ij}\theta^{kl}\partial_j\partial_l\langle\hat{A}_i,\hat{A}_k\rangle_{*_2}\right]. \tag{3.8}$$

After some rearrangements of terms, this can be written:

$$\hat{S}_{DBI}\Big|_{SW} = \int \frac{\sqrt{\det(G+2\pi\alpha'\Phi)}}{G_s}\Big[1+\theta^{ij}\partial_j\hat{A}_i+\frac{1}{2}\theta^{ij}\theta^{kl}\langle\partial_j\hat{A}_k,\partial_l\hat{A}_i\rangle_{*_2}+ \\ +\frac{1}{2}\theta^{ba}\theta^{kl}\langle\hat{A}_k,\partial_l\hat{F}_{ab}\rangle_{*_2}+\frac{1}{8}\theta^{ij}\theta^{kl}\langle\hat{F}_{ji},\hat{F}_{lk}\rangle_{*_2}\Big]. \tag{3.9}$$

Now we can take the difference of eqs. (3.9) and (3.7). The prefactor in front of each expression is the same, by virtue of eq. (2.2). Apart from this factor and the integral sign, the result is:

$$\hat{S}_{DBI}\Big|_{SW} - S_{DBI}\Big|_{SW} = \frac{1}{4}\theta^{ij}\theta^{kl}\left(\langle\hat{F}_{jk},\hat{F}_{li}\rangle_{*_2}-\hat{F}_{jk}\hat{F}_{li}\right)-\frac{1}{8}\theta^{ij}\theta^{kl}\left(\langle\hat{F}_{ji},\hat{F}_{lk}\rangle_{*_2}-\hat{F}_{ji}\hat{F}_{lk}\right). \tag{3.10}$$

To the order in which we are working, we may replace $\hat{F}$ by F everywhere in this expression.

This, then, is our prediction for the correction ΔS_{DBI}, to order $(\alpha')^2$ and to quadratic order in the field strength F, after taking the Seiberg-Witten limit. We note that this is manifestly a higher-derivative correction: it vanishes for constant F, for which the $*_2$ product reduces to the ordinary product. Expanding the $*_2$ product to 4-derivative order, we find that

$$\Delta S_{DBI}\Big|_{SW} = -\frac{1}{96}\left[\theta^{ij}\theta^{kl}\theta^{mn}\theta^{rs}\,\partial_m\partial_r F_{jk}\,\partial_n\partial_s F_{li}-\frac{1}{2}\theta^{ij}\theta^{kl}\theta^{mn}\theta^{rs}\,\partial_m\partial_r F_{ji}\,\partial_n\partial_s F_{lk}\right]. \tag{3.11}$$

This prediction may now be compared with the computation reported in eq. (4.1) of ref. [15], which gives:

$$\Delta S_{DBI}\Big|_{SW} = -\frac{(2\pi\alpha')^4}{96}\Big[h^{ij}h^{kl}h^{mn}h^{rs}\,\partial_m\partial_r F_{jk}\,\partial_n\partial_s F_{li}- \\ -\frac{1}{2}h^{ij}h^{kl}h^{mn}h^{rs}\,\partial_m\partial_r F_{ji}\,\partial_n\partial_s F_{lk}\Big], \tag{3.12}$$

where the matrix h^{ij} is defined as:

$$h^{ij} \equiv \left(\frac{1}{g+2\pi\alpha'(B+F)}\right)^{ij}, \tag{3.13}$$

Taking the Seiberg-Witten limit, which amounts to the replacement $2\pi\alpha' h \to (1+\theta F)^{-1}\theta$, and further restricting to terms quadratic in F, we find exact agreement with eq. (3.11) above.

3.2 Graviton coupling, order F^2

In this subsection, we will compare the coupling of the bulk graviton to the energy-momentum tensor on the commutative and noncommutative sides. On the commutative side, we start again with the expression in eq. (3.1), but this time we use the full form of N as defined in eq. (2.2):

$$N^{ij} \equiv \left(\frac{1}{g + 2\pi\alpha'(B+F)}\right)^{ij} = \frac{\theta^{ij}}{2\pi\alpha'} + M^{ij} \,, \tag{3.14}$$

where

$$M^{ij} \equiv \left(\frac{1}{G + 2\pi\alpha'\Phi}\right)^{ij} . \tag{3.15}$$

As the linear coupling to the graviton starts at order $(\alpha')^2$, we now have to go beyond the leading term in the Seiberg-Witten limit. Hence we will keep terms up to order M^2.

Expanding S_{DBI} around this limit and keeping terms to order $(\alpha')^2$, and using the Seiberg-Witten map, we find:

$$\begin{aligned} S_{DBI} = \int \frac{\sqrt{\det(g+2\pi\alpha' B)}}{g_s} \Bigg[1 + & \frac{2\pi\alpha'}{2} \Big\{ M^{ji} (F_{ij} + \theta^{kl} \langle A_k, \partial_l F_{ij} \rangle_{*_2} + \theta^{kl} \langle F_{jk}, F_{li} \rangle_{*_2}) \Big\} + \\ & + \frac{(2\pi\alpha')^2}{8} \left\{ \frac{2}{2\pi\alpha'} (\mathrm{tr}\, MF)(\mathrm{tr}\, \theta F) + (\mathrm{tr}\, MF)^2 \right\} - \\ & - \frac{(2\pi\alpha')^2}{4} \left\{ \mathrm{tr}\, MFMF + \frac{2}{2\pi\alpha'} \mathrm{tr}\, MF\theta F \right\} + \\ & + \text{terms not involving } M + \text{order } F^3 \Bigg] . \end{aligned} \tag{3.16}$$

Turning now to the noncommutative action, the graviton coupling is obtained by expanding the DBI action around the Seiberg-Witten limit to order $(\alpha')^2$. There could in principle have been other relevant α' corrections to the DBI action, but these are absent by virtue of the result in refs. [17, 18] that the energy-momentum tensor as calculated from string amplitudes agrees with the one obtained by just expanding the DBI action to this order. Thus we have, in momentum space:

$$\begin{aligned} \hat{S}_{DBI} &= \frac{1}{G_s} \int L_* \left[\sqrt{\det(G + 2\pi\alpha'(\hat{F} + \Phi))}\, W(x,C) \right] * e^{ik.x} \\ &= \frac{1}{G_s} \int \sqrt{\det G}\, L_* \left[\left(1 - \frac{1}{4} (2\pi\alpha')^2\, \mathrm{tr}\, G^{-1} (\hat{F} + \Phi) G^{-1} (\hat{F} + \Phi) \right) W(x,C) \right] \star \\ &\quad \star e^{ik.x} + \cdots . \end{aligned} \tag{3.17}$$

The piece of the above expression that is order 1 in α' has already been computed earlier for the dilaton coupling. It contributes to the coupling of the trace of the graviton. The new nontrivial coupling is given by the order $(\alpha')^2$ term.

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To compare with the commutative side, it is convenient to expand the above action differently, in terms of M rather than G. We get:

$$\begin{aligned}\hat{S}_{DBI} = \frac{1}{G_s}\int \sqrt{\det(G+2\pi\alpha'\Phi)}\,\times \\ \times\Bigg[L_*(W(x,C)) + \frac{2\pi\alpha'}{2}\Big\{\operatorname{tr} MF + M^{kl}\theta^{ij}\langle\partial_j F_{lk}, A_i\rangle_{*_2} + \frac{1}{2}\langle \operatorname{tr} MF, \operatorname{tr}\theta F\rangle_{*_2}\Big\} - \\ - \frac{(2\pi\alpha')^2}{4}\operatorname{tr}\langle MF, MF\rangle_{*_2} + \frac{(2\pi\alpha')^2}{8}\langle \operatorname{tr} MF, \operatorname{tr} MF\rangle_{*_2} + \cdots\Bigg]. \qquad (3.18)\end{aligned}$$

Now taking the difference of the noncommutative and commutative actions in eqs. (3.18) and (3.16), and expanding the result to 4-derivative order, we get the prediction:

$$\begin{aligned}\Delta S_{DBI}\Big|_{SW} = -\frac{2\pi\alpha'}{48}\Big\{ M^{ij}\theta^{kl}\theta^{mn}\theta^{rs}\partial_m\partial_r F_{jk}\partial_n\partial_s F_{li} - \\ -\frac{1}{2}M^{ij}\theta^{kl}\theta^{mn}\theta^{rs}\partial_m\partial_r F_{ji}\partial_n\partial_s F_{lk}\Big\} - \\ -\frac{(2\pi\alpha')^2}{96}\Big\{ M^{ij}M^{kl}\theta^{mn}\theta^{rs}\partial_m\partial_r F_{jk}\partial_n\partial_s F_{li} - \\ -\frac{1}{2}M^{ij}M^{kl}\theta^{mn}\theta^{rs}\partial_m\partial_r F_{ji}\partial_n\partial_s F_{lk}\Big\}. \qquad (3.19)\end{aligned}$$

Note that contrary to appearances, both of the above terms are of order $(\alpha')^2$. This is because if one inserts G in place of M in the first line, the result vanishes.

The above can now be compared with the result of ref. [15] quoted above in eq. (3.12). Here one has to insert $h \sim \theta/2\pi\alpha' + M$ which is true after we neglect the F in the denominator of h (see eqs. (3.13), (2.2)). We see that eq. (3.19) is reproduced perfectly if one retains only the term proportional to $\theta^{mn}\theta^{rs}$ from $h^{mn}h^{rs}$, while keeping the terms proportional to $M^{ij}\theta^{kl}$ and $M^{ij}M^{kl}$ in $h^{ij}h^{kl}$.

One can, however, keep factors of M in the expansion of $h^{mn}h^{rs}$, and this leads to other terms from eq. (3.12) that are not reproduced by our computations. These terms are comparable to curvature couplings in that they are linear or quadratic in the metric, and quadratic in derivatives. Since the computations of ref. [15] were performed in flat space neglecting the presence of curvature couplings, it is perhaps not surprising that we do not find agreement for those terms. We hope to return to this point in the future.

4. Chern-Simons action

In this section we compare the Chern-Simons actions in the commutative and noncommutative descriptions. The first such comparison is that of the coupling to the

10-form RR potential $C^{(10)}$. In this case we have, in momentum space,

$$\begin{aligned} S_{CS} &= \frac{1}{g_s}\widetilde{C}^{(10)}(-k)\,\delta(k)\hat{S}_{CS} \\ &= \frac{1}{g_s}\widetilde{C}^{(10)}(-k)\int L_*\left\{\frac{\mathrm{Pf}\,Q}{\mathrm{Pf}\,\theta}W(x,C)\right\} * e^{ik.x}\,. \end{aligned} \tag{4.1}$$

In this case, it has been argued [15] that $\Delta S_{CS} = 0$, so the two expressions above agree exactly, leading to the topological identity of refs. [11, 12, 13]. In these papers it was also shown that an analogous result holds for comparison of the coupling to the 8-form RR potential $C^{(8)}$, leading to an exact expression for the Seiberg-Witten map in the abelian case.

For the coupling to the RR forms $C^{(6)}, C^{(4)}, C^{(2)}$ and $C^{(0)}$, there are in general α' corrections involving derivatives of the field strength. A subset of these has been computed explicitly in ref. [15]. We will parametrize these derivative corrections as follows:

$$S_{CS} + \Delta S_{CS} = \frac{1}{g_s}\int \sum_n C^{(n)} \wedge e^{2\pi\alpha'(B+F)} \wedge e^{W_4+W_6+W_8+W_{10}}\,, \tag{4.2}$$

where W_{2n} are $2n$-forms made out of F and its derivatives, containing explicit powers of α'. The expression on the r.h.s. is to be expanded and then the forms of total dimension 10 are kept. This parametrization is inspired by the lowest order computations in ref. [15], which we will confirm using noncommutativity, and which give rise to rather simple expressions for the W_{2n}. However, it is important to keep in mind that eq. (4.2) is a general parametrization. The results that one finds for derivative corrections can always be cast in this form. A point to note here is that our notation is not identical to that of ref. [15], for example what is called W_8 there is the sum of our W_8 and $\frac{1}{2}W_4 \wedge W_4$.

4.1 4-form corrections, order F^2

We turn now to the 4-form that couples to $C^{(6)}$. The commutative Chern-Simons coupling in this case is proportional to $(B+F)\wedge(B+F)$. Expanding this leads to three terms, proportional to $B\wedge B$, $B\wedge F$ and $F\wedge F$. It is easy to see that on the noncommutative side too there are three terms [10], which can be respectively matched with these three. Matching of $B\wedge B$ leads to the topological identity which was already discovered by examining the RR 10-form coupling. Matching $B\wedge F$ similarly leads to the Seiberg-Witten map. Hence the only new information comes from matching $F\wedge F$, from which we will learn about derivative corrections. This pattern will be repeated when we study lower RR forms. Therefore at each stage, it suffices to examine the F^n part of the CS coupling.

Hence the Chern-Simons coupling that we will now study (an overall factor of $(2\pi\alpha')^2$ has been removed) is

$$S_{CS} = \frac{1}{g_s}\int C^{(6)} \wedge \left(\frac{1}{2} F \wedge F\right). \tag{4.3}$$

According to our parametrization, the correction ΔS_{CS} is of the form:

$$\Delta S_{CS} = \frac{1}{g_s}\int C^{(6)} \wedge W_4\,, \tag{4.4}$$

where W_4 is a 4-form. This was computed to 4-derivative order, or equivalently order $(\alpha')^2$, in ref. [15], where it was found to be:

$$W_4 = (2\pi\alpha')^2\,\frac{\zeta(2)}{8\pi^2}\,\mathrm{tr}(h\mathbf{S}\wedge h\mathbf{S}) + \cdots\,. \tag{4.5}$$

The 2-form $\mathbf{S}_{ij}$ in the above expression is defined by

$$\mathbf{S}_{ij} \equiv \tfrac{1}{2} S_{ij,ab}\, dx^a \wedge dx^b \equiv \frac{1}{2}\left(\partial_i\partial_j F_{ab} + (2\pi\alpha')\, 2h^{cd}\,\partial_i F_{ac}\,\partial_j F_{db}\right) dx^a\wedge dx^b \tag{4.6}$$

and contractions are carried out using h, defined in eq. (3.13) above.

In the Seiberg-Witten limit, $h^{ij} \to \theta/2\pi\alpha'$ and this correction becomes

$$\Delta S_{CS}\Big|_{SW} = \frac{1}{g_s}\int C^{(6)}\wedge\frac{1}{48}\,\mathrm{tr}(\theta\mathbf{S}\wedge\theta\mathbf{S})\,, \tag{4.7}$$

where we have inserted $\zeta(2) = \pi^2/6$.

Now let us work in the limit of small field strength, keeping only the leading (in this case quadratic) terms as $F \to 0$, and test whether eq. (2.11) indeed reproduces this term. For the 4-form correction, the operative term in eq. (2.11) (again with an overall $(2\pi\alpha')^2$ removed) is:

$$\frac{1}{2}\int L_*\left[\mathrm{Pf}(1-\theta\hat{F})\left(\hat{F}\frac{1}{1-\theta\hat{F}}\right)\wedge\left(\hat{F}\frac{1}{1-\theta\hat{F}}\right)W(x,C)\right] * e^{ik.x}\,. \tag{4.8}$$

Since we are working to order F^2, we can neglect the difference between F and $\hat{F}$, and also the effect of the pfaffian, the $(1-\theta\hat{F})$ denominators, and the Wilson line. Indeed, the only effect of noncommutativity that we need to keep is the fact that the L_* prescription leads to $*_n$ products, in this case $*_2$. Thus the above expression reduces to

$$\frac{1}{2}\int \langle F\wedge F\rangle_{*_2}\, e^{ik.x}\,. \tag{4.9}$$

In the small-F limit we can also reduce the 2-form $\mathbf{S}$ in eq. (4.6) to

$$S_{ij,ab} \sim \partial_i\partial_j F_{ab}\,. \tag{4.10}$$

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As a result, eq. (2.14) tells us that we should find:

$$\frac{1}{2}F\wedge F+\frac{1}{48}\,\mathrm{tr}(\theta\mathbf{S}\wedge\theta\mathbf{S})=\frac{1}{2}\langle F\wedge F\rangle_{*_2} \tag{4.11}$$

(for the $*_n$ product of n differential forms, we use the notation $\langle f_1\wedge f_2\wedge\cdots f_n\rangle_{*_n}$).

It is easy to check, from the definition of the $*_2$ product in eq. (2.12), that:

$$\frac{1}{2}\langle F\wedge F\rangle_{*_2}=\frac{1}{2}F\wedge F+\frac{1}{48}\,\mathrm{tr}(\theta^{ij}\partial_j\partial_k F\wedge\theta^{kl}\partial_l\partial_m F)+\cdots \tag{4.12}$$

in agreement with the l.h.s. of eq. (4.11).

In this discussion of 4-form corrections, we have so far restricted our attention to 4-derivative terms that are quadratic in F. Let us now go beyond the 4-derivative approximation but retain the restriction to quadratic F (thus, F is small but not slowly varying). In this case, following the techniques of ref. [15], one could explicitly compute higher-order corrections in α' to eq. (4.5). This has not been actually done so far, to the best of our knowledge. But from our considerations, we can predict what the result will be in the Seiberg-Witten limit, to every order in derivatives! Indeed, our prediction amounts to the statement that to quadratic order in F,

$$\Delta S_{CS}\Big|_{SW}=\frac{1}{g_s}\int C^{(6)}\wedge\left\{\frac{1}{2}\langle F\wedge F\rangle_{*_2}-\frac{1}{2}F\wedge F\right\}, \tag{4.13}$$

where the r.h.s. has infinitely many higher-derivative terms.

For example, the 8-derivative correction arising out of this is:

$$\Delta S_{CS}\Big|_{SW}=\frac{1}{g_s}\int C^{(6)}\wedge\left\{\frac{1}{3840}\,\theta^{ij}\theta^{kl}\theta^{mn}\theta^{pq}\,\partial_i\partial_k\partial_m\partial_p F\wedge\partial_j\partial_l\partial_n\partial_q F\right\} \tag{4.14}$$

and this should be checked by explicit computation of string amplitudes.

It is tempting to speculate that one can read off the result even away from the Seiberg-Witten limit, by making the substitution

$$\theta^{ij}\to 2\pi\alpha' N^{ij}=2\pi\alpha'\left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}. \tag{4.15}$$

The problem is that this substitution is not unique. The l.h.s. is antisymmetric, so there could be terms that are nonvanishing in general but vanish in the SW limit. If so, we would not find them by our procedure. Nevertheless, if the above substitution turns out to make sense, it would amount to saying that the 4-form corrections to all derivative orders, but quadratic in F, are encoded in a $*_2$ product whose noncommutativity parameter is $2\pi\alpha' h$ (a matrix of no definite symmetry) rather than θ. This is suggestive of a beautiful mathematical structure underlying stringy α' corrections.

4.2 6-form corrections, order F^3

Let us now look at corrections to the 6-form that couples to the RR 4-form potential $C^{(4)}$. We continue to work in the limit of small F, so we only keep the lowest power of F, in this case F^3, in all terms. The basic Chern-Simons coupling of interest in this subsection is:

$$S_{CS} = \frac{1}{g_s}\int C^{(4)} \wedge \left(\frac{1}{3!}F\wedge F\wedge F\right) \tag{4.16}$$

and the correction this time is parametrized as:

$$\Delta S_{CS} = \frac{1}{g_s}\int C^{(4)} \wedge (F\wedge W_4 + W_6)\,. \tag{4.17}$$

Here W_4 is given to 4-derivative order in eq. (4.5), and W_6 has been determined by explicit computation[15] to be:

$$W_6 = (2\pi\alpha')^3\,\frac{\zeta(3)}{24\pi^3}\,\mathrm{tr}(h\mathbf{S}\wedge h\mathbf{S}\wedge h\mathbf{S}) + \cdots \tag{4.18}$$

to 6-derivative order. Following the same arguments as for the 4-form case, and restricting to the leading terms of cubic order in F, we expect to find that

$$\frac{1}{3!}F\wedge F\wedge F + F\wedge W_4\Big|_{SW} + W_6\Big|_{SW} = \frac{1}{3!}\langle F\wedge F\wedge F\rangle_{*_3}\,. \tag{4.19}$$

We immediately seem to face a problem. For even integer arguments we have the property that $\zeta(n)/\pi^n$ is rational, but for odd integer arguments there is no such property. Hence there does not seem to be any way to obtain a number like $\zeta(3)$ by expanding $*_3$. Fortunately, in the Seiberg-Witten limit, W_6 to the order given in eq. (4.18) vanishes. This is because, in this limit, h is replaced by θ, whose antisymmetry together with the symmetry of $\mathbf{S}$ ensures that the trace in eq. (4.18) is zero. There is still something to check, however. We have already seen that

$$W_4\Big|_{SW} = \frac{1}{2}\langle F\wedge F\rangle_{*_2} - \frac{1}{2}F\wedge F\,. \tag{4.20}$$

Thus eq. (4.19) implies the identity:

$$\frac{1}{3!}F\wedge F\wedge F + F\wedge\left(\frac{1}{2}\langle F\wedge F\rangle_{*_2} - \frac{1}{2}F\wedge F\right) + W_6\Big|_{SW} = \frac{1}{3!}\langle F\wedge F\wedge F\rangle_{*_3} \tag{4.21}$$

which determines the Seiberg-Witten limit of W_6 entirely in terms of $*_n$ products.

$$W_6\Big|_{SW} = \frac{1}{3!}\langle F\wedge F\wedge F\rangle_{*_3} - \frac{1}{2}F\wedge\langle F\wedge F\rangle_{*_2} + \frac{1}{3}F\wedge F\wedge F\,. \tag{4.22}$$

Since we know that the l.h.s. vanishes to 6-derivative order, it must be the case that the r.h.s. is also zero to this order (in particular, the 4-derivative terms cancel out), which one can confirm by expanding $*_3$ and $*_2$.

4.3 8-form corrections, order F^4

This case is important because the explicit computation of derivative corrections to the Chern-Simons action produces a new 8-form W_8, that starts with 8 derivatives. The computed term is nonvanishing even in the Seiberg-Witten limit. Thus we have a new numerical coefficient and index structure to compare with the predictions of noncommutativity. In this subsection we neglect all terms that are higher order in F compared to the leading power F^4.

In this case, the derivative corrections to the coupling

$$\frac{1}{g_s}\int C^{(2)}\wedge\left(\frac{1}{4!}F\wedge F\wedge F\wedge F\right) \tag{4.23}$$

are parametrized as:

$$\Delta S_{CS}=\frac{1}{g_s}\int C^{(2)}\wedge\left(\frac{1}{2}F\wedge F\wedge W_4+\frac{1}{2}W_4\wedge W_4+F\wedge W_6+W_8\right). \tag{4.24}$$

Here W_4 and W_6 have already been determined, while W_8 has been computed to 8-derivative order [15], yielding:

$$W_8=(2\pi\alpha')^4\,\frac{\zeta(4)}{64\pi^4}\,\mathrm{tr}(h\mathbf{S}\wedge h\mathbf{S}\wedge h\mathbf{S}\wedge h\mathbf{S})+\cdots. \tag{4.25}$$

We note that $\zeta(4)=\pi^4/90$ so the numerical coefficient is indeed a rational number. Moreover, the above expression, like that for W_4, does not vanish in the Seiberg-Witten limit.

Hence repeating the arguments of the previous sections, our prediction is that to 4th order in F:

$$\begin{aligned}\frac{1}{4!}F\wedge F\wedge F\wedge F+\frac{1}{2}F\wedge F\wedge W_4\big|_{SW}+F\wedge W_6\big|_{SW}+\frac{1}{2}W_4\wedge W_4\big|_{SW}+W_8\big|_{SW}&=\\ =\frac{1}{4!}\langle F\wedge F\wedge F\wedge F\rangle_{*_4}\,.&\end{aligned} \tag{4.26}$$

Using eqs. (4.20) and (4.22) for W_4 and W_6 in the Seiberg-Witten limit, we get:

$$\begin{aligned}W_8\big|_{SW}=&\ \frac{1}{4!}\langle F\wedge F\wedge F\wedge F\rangle_{*_4}-\frac{1}{3!}F\wedge\langle F\wedge F\wedge F\rangle_{*_3}-\frac{1}{8}\langle F\wedge F\rangle_{*_2}\langle F\wedge F\rangle_{*_2}+\\ &+\frac{1}{2}F\wedge F\wedge\langle F\wedge F\rangle_{*_2}-\frac{1}{4}F\wedge F\wedge F\wedge F\,.\end{aligned} \tag{4.27}$$

It is a tedious but straightforward exercise to expand the right hand side in powers of derivatives. At the end of it, one finds that, to 8-derivative order,

$$W_8\big|_{SW}=\frac{1}{5760}\,\mathrm{tr}(\theta\mathbf{S}\wedge\theta\mathbf{S}\wedge\theta\mathbf{S}\wedge\theta\mathbf{S}) \tag{4.28}$$

in perfect agreement with the Seiberg-Witten limit of eq. (4.25). Note that this computation not only predicts the correct 8-derivative term that appears in $W_8\big|_{SW}$, with

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the correct coefficient, but also involves a number of delicate cancellations between different terms on the right hand side. These cancellations involve both 4-derivative and 8-derivative terms, and are crucial in ensuring that the surviving 8-derivative term has precisely the index contractions required to match with eq. (4.25).

4.4 4-form corrections, higher orders in F

In this subsection we return to the 4-derivative, 4-form corrections that were examined in subsection 4.1, but now we relax the requirement that F is small. Thus we have to keep higher orders in F. Since the noncommutative field strength $\hat{F}$ is an infinite series in powers of F and its derivatives, given by the Seiberg-Witten map, we will have to face this complication now. In addition, the factor $\mathrm{Pf}\, Q / \mathrm{Pf}\, \theta$ and the Wilson line will all make contributions. To keep things manageable, we restrict our attention to terms involving only 4 derivatives and work in order F^3.

This check is very nontrivial because, in the Chern-Simons context, it involves for the first time all the different contributions in eq. (4.8). Since two explicit $\hat{F}$ factors are already present, to get a third one we can expand either the pfaffian, or the $(1-\theta\hat{F})$ denominators, or the Wilson line, in each case to first order. Also, in the second order term in $\hat{F}$ we must insert the Seiberg-Witten map to the lowest nontrivial order, which leads to more F^3 terms.

The computation consists of adding together the following terms. For convenience, we write out the 4-form indices a, b, c, d explicitly, and it is to be understood that they are totally antisymmetrized. The first contribution is:

$$\hat{F}^2 \text{ term}: \quad \frac{1}{2}\langle \hat{F}_{ab}, \hat{F}_{cd}\rangle_{*_2} = \frac{1}{2}F_{ab}F_{cd} - \theta^{ij}\langle\, \langle A_i, \partial_j F_{ab}\rangle_{*_2} - \langle F_{ai}, F_{bj}\rangle_{*_2}, F_{cd}\rangle_{*_2}\,, \tag{4.29}$$

where the Seiberg-Witten map has been inserted on the r.h.s. For the rest, we get

$$\begin{aligned} (1-\theta\hat{F})\text{ denominators}: &\quad \theta^{ij}\langle \hat{F}_{ai}, \hat{F}_{jb}, \hat{F}_{cd}\rangle_{*_3} \\ \text{pfaffian}: &\quad -\frac{1}{4}\theta^{ij}\langle \hat{F}_{ji}, \hat{F}_{ab}\hat{F}_{cd}\rangle_{*_3} \\ \text{Wilson line}: &\quad \frac{1}{2}\theta^{ij}\partial_j\langle \hat{A}_i, \hat{F}_{ab}, \hat{F}_{cd}\rangle_{*_3}\,. \end{aligned} \tag{4.30}$$

In these terms, we can replace $\hat{F}$ by F everywhere since we are working to order F^3.

As a first check, it is easy to see that on replacing all $*$ products by ordinary products, all the cubic terms add up to zero. This amounts to the fact that there are no corrections to this Chern-Simons term that is 0-derivative but cubic in F.

Now we proceed to expand the $*_2$ and $*_3$ products, keeping terms with upto 4 derivatives. The relevant formulae are:

$$\begin{aligned} \langle f, g\rangle_{*_2} &\sim fg - \frac{1}{24}\theta^{pr}\theta^{qs}\partial_p\partial_q f\, \partial_r\partial_s g \\ \langle f, g, h\rangle_{*_3} &\sim fgh - \frac{1}{24}\theta^{pr}\theta^{qs}\Big(f\partial_p\partial_q g\partial_r\partial_s h + g\partial_p\partial_q h\partial_r\partial_s f + h\partial_p\partial_q f\partial_r\partial_s g\Big). \end{aligned} \tag{4.31}$$

Using this expansion and summing up eqs.(4.29) and (4.30), the final result for the cubic terms is then:

$$-\frac{1}{12}\theta^{ij}\theta^{pr}\theta^{qs}\,\partial_p F_{ai}\,\partial_q F_{bj}\,\partial_r\partial_s F_{cd}+\frac{1}{24}\theta^{ij}\theta^{pr}\theta^{qs}\,F_{pi}\,\partial_q\partial_j F_{ab}\,\partial_r\partial_s F_{cd}\,. \tag{4.32}$$

This is to be compared with the results of explicit computation.

From eq. (4.5), we see that W_4 contains two types of F^3 terms. One comes from inserting the second term in eq. (4.6) into eq. (4.5). Another arises by keeping the linear terms in eq. (4.6), but noting that h^{ij} in eq. (3.13) contains powers of F. In the Seiberg-Witten limit this gives us

$$2\pi\alpha' h^{ij}\to\left(\frac{1}{B+F}\right)^{ij}\sim(\theta-\theta F\theta)^{ij}+\cdots\,. \tag{4.33}$$

The term linear in F above then gives the second contribution to the F^3 terms.

We therefore find that the 4-derivative contribution to W_4 of order F^3, in the Seiberg-Witten limit, is made up of the following two terms:

$$\Big[W_4(\text{order }F^3)\Big]_{abcd}=\frac{1}{12}\theta^{ij}\theta^{kl}\theta^{pq}\partial_j\partial_k F_{ab}\partial_l F_{cp}\,\partial_i F_{dq}+\frac{1}{24}\theta^{ij}\theta^{kl}\theta^{pq}F_{ki}\partial_p\partial_j F_{ab}\,\partial_l\partial_q F_{cd}\,, \tag{4.34}$$

where again we have displayed the form indices a,b,c,d explicitly, and antisymmetrization over them on the r.h.s. is understood. Comparing eqs.(4.34) and (4.32), we see after rearranging a few indices that they agree perfectly. This once more demonstrates that the Seiberg-Witten limit of derivative corrections in ordinary string theory can be determined just using noncommutativity.

It should be straightforward to extend the above procedure to 4-derivative terms of order F^4 and higher, and compare them with the relevant results in ref. [15], though we will not do this here.

5. Conclusions

The amazing agreement between our calculations and the boundary-state computations performed by Wyllard[15] calls for some comment. This agreement basically stems from the fact that the variables used in ref. [15] are, in a precise sense, the correct ones in terms of which a comparison can be made. Indeed, it is a field redefinition performed in ref. [15, eq. (2.15)] that plays the crucial role in ensuring this agreement. The motivation for this field redefinition was to eliminate derivative corrections to the coupling to the RR 8-form $C^{(8)}$. Because of this, the conventional form of the Seiberg-Witten map holds for the *same* choice of variables in which the results of ref. [15] are expressed, as was seen in refs. [11, 12, 13]. Once this is ensured, the variables are completely determined and there is no longer an ambiguity of field redefinitions.

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To summarize, in this paper we have demonstrated that noncommutativity is a powerful tool in determining an infinite set of stringy α' corrections to the ordinary (commutative) D-brane effective action, including couplings to closed-string backgrounds. This works basically because the insertion of Wilson lines ensures the exact equivalence of commutative and noncommutative actions, and because the Seiberg-Witten limit drastically simplifies the noncommutative description while retaining higher-derivative α' corrections on the commutative side.

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